DRAFT COPY

Truncation Error Analysis for ADCIRC

K. M. Dresback and R. L. Kolar

Environmental Modeling/GIS Laboratory (EM/GIS)
School of Civil Engineering and Environmental Science
University of Oklahoma
Norman, OK 73019

Technical Report No. 04-02

(Keywords: shallow water modeling, GWC equation, finite elements, NCM equation, truncation errors, CM equation)

Abstract

Herein, we examine the truncation errors for the governing equations of ADCIRC (GWC, NCM and CM equations). To evaluate the truncation errors, we utilize Taylor series expansions for all the terms in the governing equations. By using the Taylor series, we expand the dependent variables around a common point for evaluation purposes. After using the Taylor series to expand out the dependent variables in the equations, the results are subtracted from the continuous equations in order to obtain the truncation error. The discrete form of the equations come from using C^0 *linear finite element for the spatial discretization and a two time level scheme centered at* $k+1/2$ *for the temporal discretization (a Crank-Nicholson scheme). The nonlinear terms in the equations employ an explicit formulation. We utilize the exact quadrature rules and averaged terms are based on an elemental average. In the evaluation of the truncation errors, we employ Mathematica® to expand the Taylor Series to the seventh order terms; however, we report errors only to the first two orders of errors. From these truncation errors, we determined that the GWC equation is first-order accurate in time if the advective terms are in non-conservative form, while it is second-order accurate in time if the advective terms are in conservative form. In space, the GWC equation is first-order accurate for variable spacing; while, it is second-order accurate for constant spacing. For the NCM and CM equations, we found that they are first-order accurate in time and space if we use variable spacing while it is second order accurate in space if we use constant spacing. Also, both momentum equations become second-order accurate in time if the equations are linearized.*

DRAFT COPY

Truncation Errors for the Generalized Wave Continuity Equation

First Term - $(\partial^2\zeta)/(\partial t^2)$

$$-\frac{1}{12}\,z^{(0,4)}(j,k)\,dt^2+\frac{1}{36}\,(dx(j)-dx(j+1))\,z^{(1,4)}(j,k)\,dt^2-\frac{1}{72}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(2,4)}(j,k)\,dt^2+\frac{1}{3}\,(dx(j)-dx(j+1))\,z^{(1,2)}(j,k)-\frac{1}{6}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(2,2)}(j,k)$$

Second Term - $G((\partial\zeta)/(\partial t))$

$$-\frac{1}{6}\,G\,z^{(0,3)}(j,k)\,dt^2+\frac{1}{18}\,G\,(dx(j)-dx(j+1))\,z^{(1,3)}(j,k)\,dt^2-\frac{1}{36}\,G\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(2,3)}(j,k)\,dt^2+\frac{1}{3}\,G\,(dx(j)-dx(j+1))\,z^{(1,1)}(j,k)-\frac{1}{6}\,G\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(2,1)}(j,k)$$

Finite Amplitude Term - part 1 - $gh((\partial^2\zeta)/(\partial x^2))$

$$\frac{1}{3}\,g\,h^{(1,0)}(j,k)\,z^{(1,2)}(j,k)\,dt^2+\frac{1}{6}\,g\,(dx(j+1)-dx(j))\,z^{(1,2)}(j,k)\,h^{(2,0)}(j,k)\,dt^2+\frac{1}{3}\,g\,h(j,k)\,z^{(2,2)}(j,k)\,dt^2+\frac{1}{6}\,g\,(dx(j+1)-dx(j))\,h^{(1,0)}(j,k)\,z^{(2,2)}(j,k)\,dt^2+\frac{1}{12}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h^{(2,0)}(j,k)\,z^{(2,2)}(j,k)\,dt^2+\frac{1}{18}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,2)}(j,k)\,h^{(3,0)}(j,k)\,dt^2+\frac{1}{9}\,g\,(dx(j+1)-dx(j))\,h(j,k)\,z^{(3,2)}(j,k)\,dt^2+\frac{1}{18}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h^{(1,0)}(j,k)\,z^{(3,2)}(j,k)\,dt^2+\frac{1}{36}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h(j,k)\,z^{(4,2)}(j,k)\,dt^2+\frac{1}{2}\,g\,(dx(j+1)-dx(j))\,z^{(1,0)}(j,k)\,h^{(2,0)}(j,k)+\frac{1}{2}\,g\,(dx(j+1)-dx(j))\,h^{(1,0)}(j,k)\,z^{(2,0)}(j,k)+\frac{1}{4}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h^{(2,0)}(j,k)\,z^{(2,0)}(j,k)+\frac{1}{6}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,0)}(j,k)\,h^{(3,0)}(j,k)+\frac{1}{3}\,g\,(dx(j+1)-dx(j))\,h(j,k)\,z^{(3,0)}(j,k)+\frac{1}{6}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h^{(1,0)}(j,k)\,z^{(3,0)}(j,k)+\frac{1}{12}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h(j,k)\,z^{(4,0)}(j,k)$$

Finite Amplitude Term - part 2 - $g((\partial^2(\zeta^2))/(\partial x^2))$

$$\frac{1}{6}\,g\,(dx(j+1)-dx(j))\,zsq^{(3,0)}(j,k)+\frac{1}{24}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,zsq^{(4,0)}(j,k)$$

Advective Term - conservative form - $(\partial^2(qu))/(\partial x^2)$

$$\frac{1}{3}\,(dx(j+1)-dx(j))\,qu^{(3,0)}(j,k)+\frac{1}{12}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,qu^{(4,0)}(j,k)$$

DRAFT COPY

Advective Term - non-conservative form part 1 - $\partial(u((\partial\zeta)/\partial t))/(\partial x)$

$$-\frac{1}{6} z^{(0,3)}(j,k)\,u^{(1,0)}(j,k)\,dt^2-\frac{1}{6} u(j,k)\,z^{(1,3)}(j,k)\,dt^2+\frac{1}{12}(dx(j)-dx(j+1))\,u^{(1,0)}(j,k)\,z^{(1,3)}(j,k)\,dt^2+$$
$$\frac{1}{12}(dx(j)-dx(j+1))\,z^{(0,3)}(j,k)\,u^{(2,0)}(j,k)\,dt^2-\frac{1}{24}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,3)}(j,k)\,u^{(2,0)}(j,k)\,dt^2+$$
$$\frac{1}{12}(dx(j)-dx(j+1))\,u(j,k)\,z^{(2,3)}(j,k)\,dt^2-\frac{1}{24}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(1,0)}(j,k)\,z^{(2,3)}(j,k)\,dt^2-$$
$$\frac{1}{36}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(0,3)}(j,k)\,u^{(3,0)}(j,k)\,dt^2-$$
$$\frac{1}{36}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u(j,k)\,z^{(3,3)}(j,k)\,dt^2+\frac{1}{2} z^{(0,2)}(j,k)\,u^{(1,0)}(j,k)\,dt+\frac{1}{2} u(j,k)\,z^{(1,2)}(j,k)\,dt+$$
$$\frac{1}{4}(dx(j+1)-dx(j))\,u^{(1,0)}(j,k)\,z^{(1,2)}(j,k)\,dt+\frac{1}{4}(dx(j+1)-dx(j))\,z^{(0,2)}(j,k)\,u^{(2,0)}(j,k)\,dt+$$
$$\frac{1}{8}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,2)}(j,k)\,u^{(2,0)}(j,k)\,dt+\frac{1}{4}(dx(j+1)-dx(j))\,u(j,k)\,z^{(2,2)}(j,k)\,dt+$$
$$\frac{1}{8}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(1,0)}(j,k)\,z^{(2,2)}(j,k)\,dt+$$
$$\frac{1}{12}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(0,2)}(j,k)\,u^{(3,0)}(j,k)\,dt+$$
$$\frac{1}{12}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u(j,k)\,z^{(3,2)}(j,k)\,dt+\frac{1}{2}(dx(j)-dx(j+1))\,u^{(1,0)}(j,k)\,z^{(1,1)}(j,k)+$$
$$\frac{1}{2}(dx(j)-dx(j+1))\,z^{(0,1)}(j,k)\,u^{(2,0)}(j,k)-\frac{1}{4}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,1)}(j,k)\,u^{(2,0)}(j,k)+$$
$$\frac{1}{2}(dx(j)-dx(j+1))\,u(j,k)\,z^{(2,1)}(j,k)-\frac{1}{4}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(1,0)}(j,k)\,z^{(2,1)}(j,k)-$$
$$\frac{1}{6}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(0,1)}(j,k)\,u^{(3,0)}(j,k)-\frac{1}{6}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u(j,k)\,z^{(3,1)}(j,k)$$

Advective Term - non-conservative form part 2 - $\partial(q((\partial u)/(\partial x)))/(\partial x)$

$$-\frac{1}{2}(dx(j)-dx(j+1))\,u^{(1,0)}(j,k)\,q^{(2,0)}(j,k)+\frac{1}{4}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(2,0)}(j,k)\,q^{(2,0)}(j,k)+$$
$$\frac{1}{2}(dx(j+1)-dx(j))\,q^{(1,0)}(j,k)\,u^{(2,0)}(j,k)+\frac{1}{6}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(1,0)}(j,k)\,q^{(3,0)}(j,k)+$$
$$\frac{1}{3}(dx(j+1)-dx(j))\,q(j,k)\,u^{(3,0)}(j,k)+\frac{1}{6}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(1,0)}(j,k)\,u^{(3,0)}(j,k)+$$
$$\frac{1}{12}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q(j,k)\,u^{(4,0)}(j,k)$$

GWC Flux Terms - $(G-\tau)((\partial q)/(\partial x))$

$$\frac{1}{2}(G-tau)(dx(j)-dx(j+1))\,q^{(2,0)}(j,k)-\frac{1}{6}(G-tau)(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(3,0)}(j,k)$$

Viscous Term - $\varepsilon((\partial^3\zeta)/(\partial x^2\partial t))$

$$\frac{1}{6} eddy\, z^{(2,3)}(j,k)\,dt^2+\frac{1}{18} eddy\,(dx(j+1)-dx(j))\,z^{(3,3)}(j,k)\,dt^2+$$
$$\frac{1}{72} eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(4,3)}(j,k)\,dt^2+\frac{1}{3} eddy\,(dx(j+1)-dx(j))\,z^{(3,1)}(j,k)+$$
$$\frac{1}{12} eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(4,1)}(j,k)$$

DRAFT COPY

Truncation Errors for the Non-Conservative Momentum Equation

Accumulation Term - $\partial u/\partial t$

$$-\frac{1}{6}u^{(0,3)}(j,k)\,dt^2+\frac{1}{18}(dx(j)-dx(j+1))\,u^{(1,3)}(j,k)\,dt^2-\frac{1}{36}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(2,3)}(j,k)\,dt^2-\frac{1}{2}u^{(0,2)}(j,k)\,dt+\frac{1}{6}(dx(j)-dx(j+1))\,u^{(1,2)}(j,k)\,dt-\frac{1}{12}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(2,2)}(j,k)\,dt+\frac{1}{3}(dx(j)-dx(j+1))\,u^{(1,1)}(j,k)+\frac{1}{6}dx(j)\,(dx(j+1)-dx(j))\,u^{(2,1)}(j,k)$$

Advective Term - $u(\partial u/\partial x)$

$$\frac{1}{2}(dx(j)-dx(j+1))\,u^{(1,0)}(j,k)^2-\frac{1}{2}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(2,0)}(j,k)\,u^{(1,0)}(j,k)+\frac{1}{2}(dx(j)-dx(j+1))\,u(j,k)\,u^{(2,0)}(j,k)-\frac{1}{6}(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u(j,k)\,u^{(3,0)}(j,k)$$

Bottom Friction Term - τu

$$-\frac{1}{4}\,tau\,u^{(0,2)}(j,k)\,dt^2+\frac{1}{12}\,tau\,(dx(j)-dx(j+1))\,u^{(1,2)}(j,k)\,dt^2+\frac{1}{24}\,tau\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(2,2)}(j,k)\,dt^2-\frac{1}{2}\,tau\,u^{(0,1)}(j,k)\,dt+\frac{1}{6}\,tau\,(dx(j)-dx(j+1))\,u^{(1,1)}(j,k)\,dt-\frac{1}{12}\,tau\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(2,1)}(j,k)\,dt+\frac{1}{3}\,tau\,(dx(j)-dx(j+1))\,u^{(1,0)}(j,k)-\frac{1}{6}\,tau\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(2,0)}(j,k)$$

Finite Amplitude Term - $g((\partial\zeta)/(\partial x))$

$$-\frac{1}{4}gz^{(1,2)}(j,k)\,dt^2+\frac{1}{8}g(dx(j)-dx(j+1))\,z^{(2,2)}(j,k)\,dt^2-\frac{1}{24}g(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(3,2)}(j,k)\,dt^2-\frac{1}{2}gz^{(1,1)}(j,k)\,dt+\frac{1}{4}g(dx(j)-dx(j+1))\,z^{(2,1)}(j,k)\,dt-\frac{1}{12}g(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(3,1)}(j,k)\,dt+\frac{1}{2}g(dx(j)-dx(j+1))\,z^{(2,0)}(j,k)-\frac{1}{6}g(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(3,0)}(j,k)$$

Viscous Term - $(\varepsilon/H)((\partial^2 q)/(\partial x^2))$

$$\frac{1}{4}\,eddy\,u^{(2,2)}(j,k)\,dt^2+\frac{1}{12}\,eddy\,(dx(j+1)-dx(j))\,u^{(3,2)}(j,k)\,dt^2+\frac{1}{48}\,eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(4,2)}(j,k)\,dt^2+\frac{1}{2}\,eddy\,u^{(2,1)}(j,k)\,dt+\frac{1}{6}\,eddy\,(dx(j)+dx(j+1))\,u^{(3,1)}(j,k)\,dt+\frac{1}{24}\,eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(4,1)}(j,k)\,dt+\frac{1}{3}\,eddy\,(dx(j+1)-dx(j))\,u^{(3,0)}(j,k)+\frac{1}{12}\,eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u^{(4,0)}(j,k)$$

DRAFT COPY

Truncation Errors for the Conservative Momentum Equation

Accumulation Term - $\partial q/\partial t$

$$-\frac{1}{6}\,q^{(0,3)}(j,k)\,dt^2+\frac{1}{18}\,(dx(j)-dx(j+1))\,q^{(1,3)}(j,k)\,dt^2-\frac{1}{36}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(2,3)}(j,k)\,dt^2-$$
$$\frac{1}{2}\,q^{(0,2)}(j,k)\,dt+\frac{1}{6}\,(dx(j)-dx(j+1))\,q^{(1,2)}(j,k)\,dt-\frac{1}{12}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(2,2)}(j,k)\,dt+$$
$$\frac{1}{3}\,(dx(j)-dx(j+1))\,q^{(1,1)}(j,k)-\frac{1}{6}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(2,1)}(j,k)$$

Advective Term - $(\partial(qu)/\partial x)$

$$(dx(j)-dx(j+1))\,q^{(1,0)}(j,k)\,u^{(1,0)}(j,k)-\frac{1}{2}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(2,0)}(j,k)\,u^{(1,0)}(j,k)-$$
$$\frac{1}{2}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(1,0)}(j,k)\,u^{(2,0)}(j,k)+$$
$$\frac{1}{2}\,(dx(j)-dx(j+1))\,(u(j,k)\,q^{(2,0)}(j,k)+q(j,k)\,u^{(2,0)}(j,k))-\frac{1}{6}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,u(j,k)\,q^{(3,0)}(j,k)-$$
$$\frac{1}{6}\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q(j,k)\,u^{(3,0)}(j,k)$$

Bottom Friction Term - τq

$$-\frac{1}{2}\,tau\,q^{(0,1)}(j,k)\,dt-\frac{1}{4}\,tau\,q^{(0,2)}(j,k)\,dt^2+\frac{1}{3}\,tau\,(dx(j)-dx(j+1))\,q^{(1,0)}(j,k)+\frac{1}{6}\,tau\,(dx(j)-dx(j+1))\,q^{(1,1)}(j,k)\,dt+$$
$$\frac{1}{12}\,tau\,(dx(j)-dx(j+1))\,q^{(1,2)}(j,k)\,dt^2+\frac{1}{36}\,tau\,(dx(j)-dx(j+1))\,q^{(1,3)}(j,k)\,dt^3-$$
$$\frac{1}{6}\,tau\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(2,0)}(j,k)-\frac{1}{12}\,tau\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(2,1)}(j,k)\,dt-$$
$$\frac{1}{24}\,tau\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(2,2)}(j,k)\,dt^2$$

Viscous Term - $\varepsilon((\partial^2 q)/(\partial x^2))$

$$\frac{1}{4}\,eddy\,q^{(2,2)}(j,k)\,dt^2+\frac{1}{12}\,eddy\,(dx(j+1)-dx(j))\,q^{(3,2)}(j,k)\,dt^2+$$
$$\frac{1}{48}\,eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(4,2)}(j,k)\,dt^2+\frac{1}{2}\,eddy\,q^{(2,1)}(j,k)\,dt+$$
$$\frac{1}{6}\,eddy\,(dx(j+1)-dx(j))\,q^{(3,1)}(j,k)\,dt+\frac{1}{24}\,eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(4,1)}(j,k)\,dt+$$
$$\frac{1}{3}\,eddy\,(dx(j+1)-dx(j))\,q^{(3,0)}(j,k)+\frac{1}{12}\,eddy\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,q^{(4,0)}(j,k)$$

DRAFT COPY

Finite Amplitude Term - Part 1 - $gh((\partial\zeta)/(\partial x))$

$$-\frac{1}{4}\,g\,h(j,k)\,z^{(1,2)}(j,k)\,dt^2+\frac{1}{8}\,g\,(dx(j)-dx(j+1))\,h^{(1,0)}(j,k)\,z^{(1,2)}(j,k)\,dt^2-$$
$$\frac{1}{16}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,2)}(j,k)\,h^{(2,0)}(j,k)\,dt^2+\frac{1}{8}\,g\,(dx(j)-dx(j+1))\,h(j,k)\,z^{(2,2)}(j,k)\,dt^2-$$
$$\frac{1}{16}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h^{(1,0)}(j,k)\,z^{(2,2)}(j,k)\,dt^2-$$
$$\frac{1}{24}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h(j,k)\,z^{(3,2)}(j,k)\,dt^2-\frac{1}{2}\,g\,h(j,k)\,z^{(1,1)}(j,k)\,dt+$$
$$\frac{1}{4}\,g\,(dx(j)-dx(j+1))\,h^{(1,0)}(j,k)\,z^{(1,1)}(j,k)\,dt-\frac{1}{8}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,1)}(j,k)\,h^{(2,0)}(j,k)\,dt+$$
$$\frac{1}{4}\,g\,(dx(j)-dx(j+1))\,h(j,k)\,z^{(2,1)}(j,k)\,dt-\frac{1}{8}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h^{(1,0)}(j,k)\,z^{(2,1)}(j,k)\,dt-$$
$$\frac{1}{12}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h(j,k)\,z^{(3,1)}(j,k)\,dt+\frac{1}{2}\,g\,(dx(j)-dx(j+1))\,h^{(1,0)}(j,k)\,z^{(1,0)}(j,k)-$$
$$\frac{1}{4}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,z^{(1,0)}(j,k)\,h^{(2,0)}(j,k)+\frac{1}{2}\,g\,(dx(j)-dx(j+1))\,h(j,k)\,z^{(2,0)}(j,k)-$$
$$\frac{1}{4}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h^{(1,0)}(j,k)\,z^{(2,0)}(j,k)-$$
$$\frac{1}{6}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,h(j,k)\,z^{(3,0)}(j,k)$$

Finite Amplitude Term - Part 2 - $(g/2)((\partial(\zeta^2))/(\partial x))$

$$-\frac{1}{8}\,g\,zsq^{(1,2)}(j,k)\,dt^2+\frac{1}{16}\,g\,(dx(j)-dx(j+1))\,zsq^{(2,2)}(j,k)\,dt^2-$$
$$\frac{1}{48}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,zsq^{(3,2)}(j,k)\,dt^2-\frac{1}{4}\,g\,zsq^{(1,1)}(j,k)\,dt+\frac{1}{8}\,g\,(dx(j)-dx(j+1))\,zsq^{(2,1)}(j,k)\,dt-$$
$$\frac{1}{24}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,zsq^{(3,1)}(j,k)\,dt+\frac{1}{4}\,g\,(dx(j)-dx(j+1))\,zsq^{(2,0)}(j,k)-$$
$$\frac{1}{12}\,g\,(dx(j)^2-dx(j+1)\,dx(j)+dx(j+1)^2)\,zsq^{(3,0)}(j,k)$$